

# **Site Symmetry in crystals : basis of the theory and applications for the electron and phonon states.**

**Robert A. Evarestov  
St. Petersburg State University**

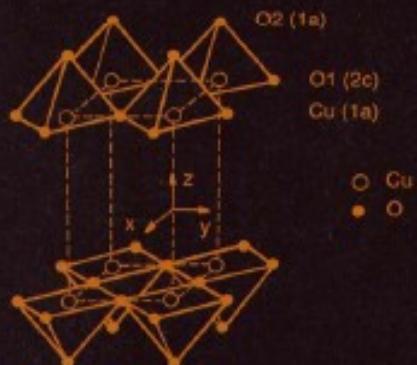
**RUSSIA**

R. A. Evarestov V. P. Smirnov

# Site Symmetry in Crystals

Theory and Applications

Second Enlarged Edition



Springer

R. A. Evarestov

# Quantum Chemistry of Solids

The LCAO  
First Principles  
Treatment  
of Crystals



Springer

Site symmetry approach establishes symmetry relations between the localized states (atomic electron states and atomic displacements) and extended molecular or crystalline states (symmetry of molecular or crystalline orbitals and phonons). The localized states transform according to the irreducible representations (irreps) of the site symmetry point groups (in crystals-point symmetry groups of Wyckoff positions occupied by atoms). The extended states transform according to the irreps of the space group of the crystal (point group of the molecule). The extended states are induced from the localized states.

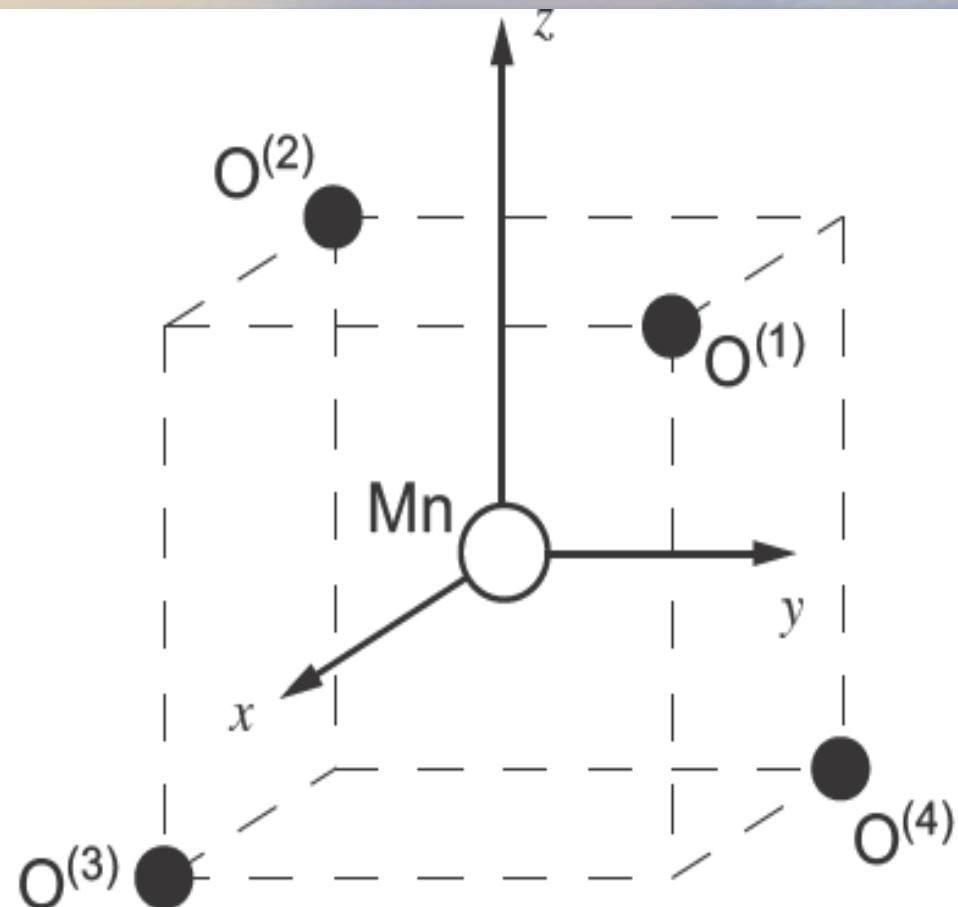


Fig. 3.8.  $\text{MnO}_4^-$  ion

$\text{KMnO}_4$  Pnma (62- $D_{2h}^{16}$ )  $Z=4$   
 4c .m.  $x 0.25 z$  8d 1  $x y z$   
 K, Mn -4c O-4c,4c,8d  
 $(\text{MnO}_4)^-$  Mn-T<sub>d</sub> O-C<sub>3v</sub>

## Point Group Tables of $T_d(-43m)$

$$G = T_d = H_{Mn} \quad H_O = C_{3v}$$

Character Table

$T_d(-43m)$	#	E	3	2	-4	m	functions
Mult.	-	1	8	3	6	6	-
$A_1$	$\Gamma_1$	1	1	1	1	1	$x^2+y^2+z^2$
$A_2$	$\Gamma_2$	1	1	1	-1	-1	-
E	$\Gamma_3$	2	-1	2	0	0	$(2z^2-x^2-y^2, x^2-y^2)$
$T_1$	$\Gamma_4$	3	0	-1	1	-1	$(J_x, J_y, J_z)$
$T_2$	$\Gamma_5$	3	0	-1	-1	1	$(x, y, z), (xy, xz, yz)$

## Point Group Tables of $C_{3v}(3m)$

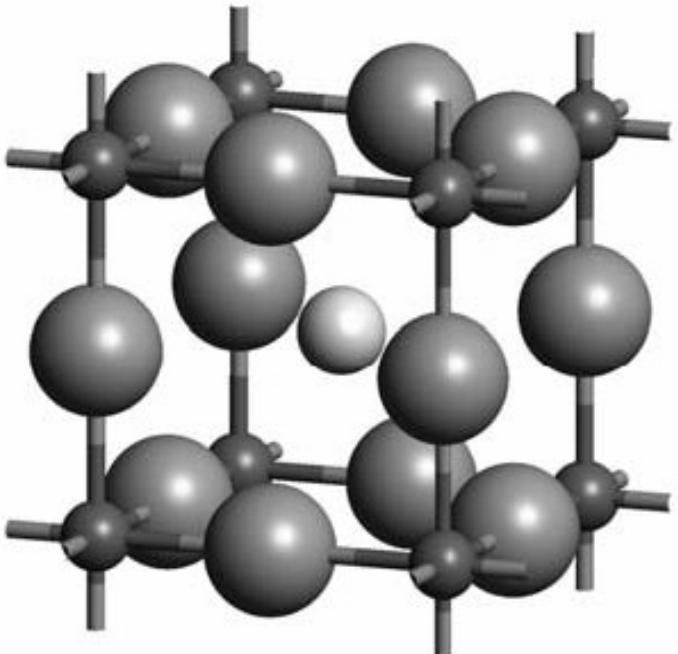
Character Table

$C_{3v}(3m)$	#	1	3	m	functions
Mult.	-	1	2	3	-
$A_1$	$\Gamma_1$	1	1	1	$z, x^2+y^2, z^2$
$A_2$	$\Gamma_2$	1	1	-1	$J_z$
E	$\Gamma_3$	2	-1	0	$(x, y), (xz, yz), (x^2-y^2, xy), (J_x, J_y)$

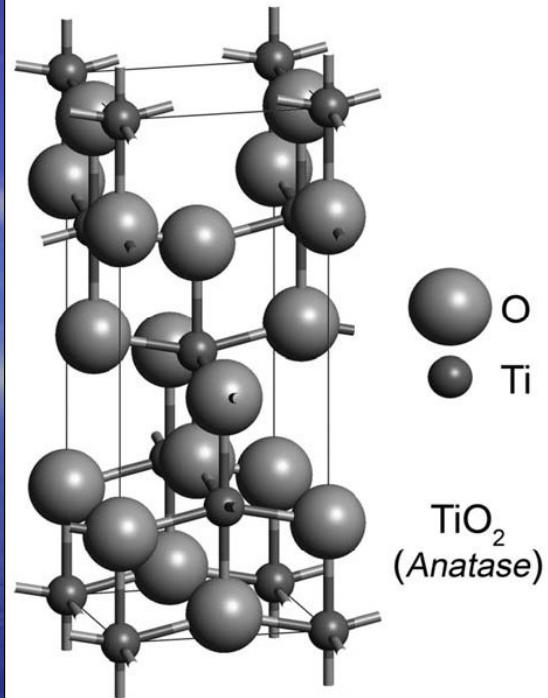
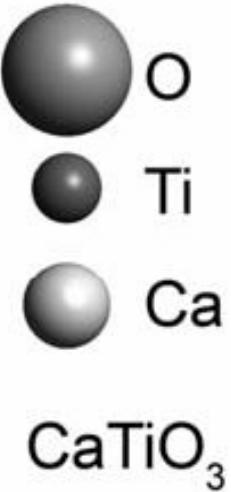
Subduced irreps of  $T_d$   
(correlation table )

$T_d$	$C_{3v}$	$C_{3v}$	$T_d$
$a_1$	$a_1$	$a_1(z)$	$a_1 t_2$
$a_2$	$a_2$	$a_2$	$a_2 a_2 t_1$
e	e	e	$e(x, y)$
$t_1$	$a_2 e$		$e t_1 t_2$
$t_2(x, y, z)$	$a_1 e$		

The Frobenius reciprocity theorem is proved [13]: the multiplicity of an irrep  $D^{(\alpha)}(g)$  of  $G$  in a rep  $d^{(\gamma)} \uparrow G$  induced by an irrep  $d^{(\gamma)}$  of  $H \subset G$  is equal to the multiplicity of an irrep  $d^{(\gamma)}$  of  $H$  in the rep  $D^{(\alpha)} \downarrow H$  subduced by  $D^{(\alpha)}$  of  $H$ .



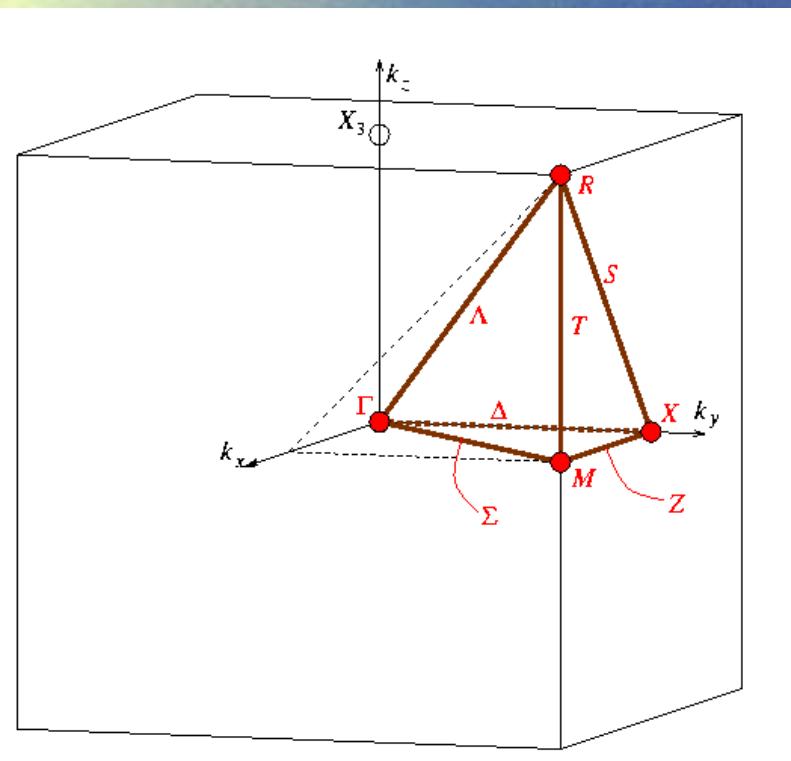
**Perovskite  $\text{CaTiO}_3$**   
 Ca 1b (0.5 ,0.5, 0.5 )  
 Ti 1a (0 ,0, 0)  
 O 3d (0.5, 0, 0; 0,0.5,0; 0, 0, 0.5)  
 (abd)=(bac)  
**Z=1, Pm-3m**  
 $(221-\text{O}_\text{h}^1)$



**Anatase  $\text{TiO}_2$**   
 two Ti atoms 2a(0 ,0 ,0; 0, 1/2, 1/4),  
 four oxygen atoms 4e (0, 0, u; 1/2, 0, -u  
 + 1/2; 0, 1/2, u +1/4;1/2, 1/2,-u+1/2).  
 The anatase structure is defined by three  
 parameters: a, c, u.  
 (a e)=(b e)  
**Z=2, I4<sub>1</sub>/amd**  
 $141-\text{D}_{4\text{h}}^{19}$

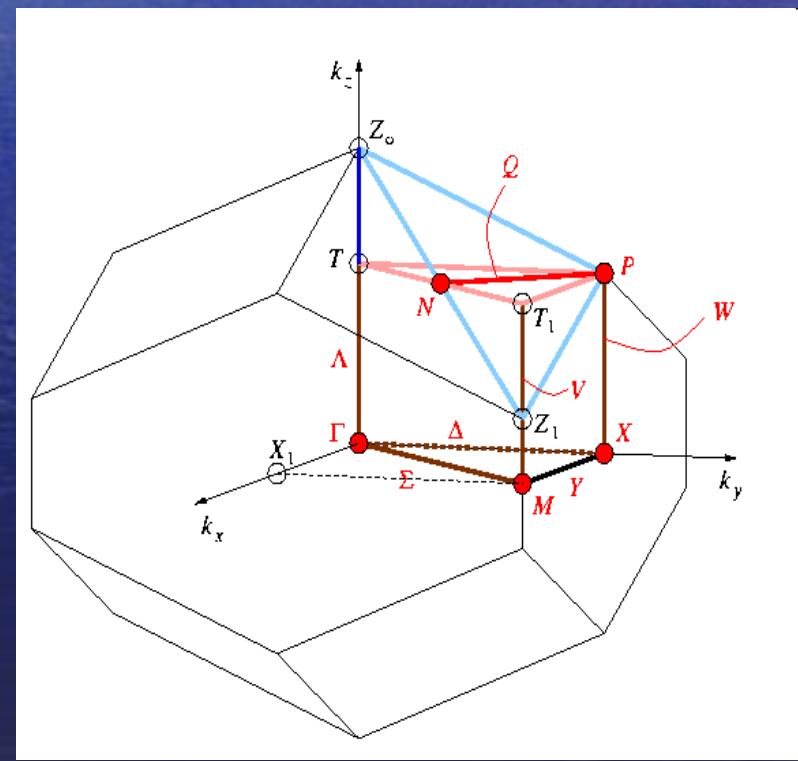
# Brillouin Zones

- Simple cubic lattice



$\Gamma(000)$   $R(0.5 0.5 0.5)$   
 $3X(0.5 0 0)$   $3M(0.5 0.5 0)$

- Tetragonal bc lattice



$\Gamma(000)$   $M(0.5 0.5 0.5)$   
2 $P(0.25 0.25 0.25)$  2 $X(0 0 0.5)$   
4 $N(0 0.5 0)$

# SYMMETRY OF LOCALIZED STATES IN PEROVSKITE AND ANATASE

## Perovskite $\text{CaTiO}_3$

Ti	a	s	$a_{1g}$
	Oh	$(x, y, z)$	$t_{1u}$
Ca	b	$(z^2, x^2-y^2)$	$e_g$

$(xy, xz, yz) \ t_{2g}$

Point Group Tables - Windows Internet Explorer

B:\60\BA009\Point Group Tables.htm

File Edit View Favorites Tools Help

@mail.ru Помощь Веб-Интернет Новый Помощь Google Москва +16°C Настройки

Google Помощь Настройки

To help protect your security, Internet Explorer has restricted this webpage from running scripts or ActiveX controls that could access your computer. Click here for options...

	$A_{1g}$	$\Gamma_1^+$	1	1	1	1	1	1	1	1	1	$x^2+y^2+z^2$
$A_{1u}$	$\Gamma_1^-$	1	1	1	1	1	-1	-1	-1	-1	-1	.
$A_{2g}$	$\Gamma_2^+$	1	-1	1	1	-1	1	-1	1	1	-1	.
$A_{2u}$	$\Gamma_2^-$	1	-1	1	1	-1	-1	1	-1	-1	1	.
$E_g$	$\Gamma_3^+$	2	0	2	-1	0	2	0	2	-1	0	$(2z^2-x^2-y^2, x^2-y^2)$
$E_u$	$\Gamma_3^-$	2	0	2	-1	0	-2	0	-2	1	0	.
$T_{2u}$	$\Gamma_5^-$	3	-1	-1	0	1	-3	1	1	0	-1	.
$T_{2g}$	$\Gamma_6^+$	3	-1	-1	0	1	3	-1	-1	0	1	$(xy, xz, yz)$
$T_{1u}$	$\Gamma_4^-$	3	1	-1	0	-1	-3	-1	1	0	1	$(x, y, z)$
$T_{1g}$	$\Gamma_4^+$	3	1	-1	0	-1	3	1	-1	0	-1	$(J_x, J_y, J_z)$

Done My Computer 100% 3:36 PM

Пуск Total Commander 7.0... Point Group Tables... EN

O	d	D <sub>4h</sub>	s,z <sup>2</sup>	a <sub>1g</sub>
			z	a <sub>2u</sub>
			x <sup>2</sup> -y <sup>2</sup>	b <sub>1g</sub>
			xy	b <sub>2g</sub>
			(xz,yz)	e <sub>g</sub>
			(x,y)	e <sub>u</sub>

Point Group Tables - Opera

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Point Group Tables rd

<http://www.cryst.ehu.es/cgi-bin/rep/programs/sa> Google 68

Find in page Find next Voice Author mode Show images Fit to width 100%

Character table

D <sub>4h</sub> (4/mmm)	#	1	2	4	2 <sub>h</sub>	2 <sub>h'</sub>	-1	m <sub>x</sub>	-4	m <sub>y</sub>	m <sub>z</sub>	functions
Mult.	-	1	1	2	2	2	1	1	2	2	2	-
A <sub>1g</sub>	$\Gamma_1^+$	1	1	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$
A <sub>2g</sub>	$\Gamma_2^+$	1	1	1	-1	-1	1	1	1	-1	-1	$J_z$
B <sub>1g</sub>	$\Gamma_3^+$	1	1	-1	1	-1	1	1	-1	1	-1	$x^2-y^2$
B <sub>2g</sub>	$\Gamma_4^+$	1	1	-1	-1	1	1	1	-1	-1	1	$xy$
E <sub>g</sub>	$\Gamma_5^+$	2	-2	0	0	0	2	-2	0	0	0	$(yz, yz)(J_x, J_y)$
A <sub>1u</sub>	$\Gamma_1^-$	1	1	1	1	1	-1	-1	-1	-1	-1	-
A <sub>2u</sub>	$\Gamma_2^-$	1	1	1	-1	-1	-1	-1	1	1	1	$z$
B <sub>1u</sub>	$\Gamma_3^-$	1	1	-1	1	-1	-1	-1	1	-1	1	-
B <sub>2u</sub>	$\Gamma_4^-$	1	1	-1	-1	1	-1	-1	1	1	-1	-

Пуск Total Com... symnlosta... Point Gro... EN 3:38 PM

Table 3.8. Simple induced representations of the  $O_h^1$  -  $Pm\bar{3}m$  space group

q	$\beta$	$\Gamma$	R		M		X	
			a	b	a	b	a	b
a	$a_{1g}$	$1^+$	$1^+$	$2^-$	$1^+$	$4^+$	$1^+$	$3^-$
	$a_{1u}$	$1^-$	$1^-$	$2^+$	$1^-$	$4^-$	$1^-$	$3^+$
	$a_{2g}$	$2^+$	$2^+$	$1^-$	$2^+$	$3^+$	$2^+$	$4^-$
	$a_{2u}$	$2^-$	$2^-$	$1^+$	$2^-$	$3^-$	$2^-$	$4^+$
	$e_g$	$3^+$	$3^+$	$3^-$	$1^+2^+$	$3^+4^+$	$1^+2^+$	$3^-4^-$
	$e_u$	$3^-$	$3^-$	$3^+$	$1^-2^-$	$3^-4^-$	$1^-2^-$	$3^+4^+$
	$t_{1g}$	$4^+$	$4^+$	$5^-$	$3^+5^+$	$2^+5^+$	$3^+5^+$	$1^-5^-$
	$t_{1u}$	$4^-$	$4^-$	$5^+$	$3^-5^-$	$2^-5^-$	$3^-5^-$	$1^+5^+$
b	$t_{2g}$	$5^+$	$5^+$	$4^-$	$4^+5^+$	$1^+5^+$	$4^+5^+$	$2^-5^-$
	$t_{2u}$	$5^-$	$5^-$	$4^+$	$4^-5^-$	$1^-5^-$	$4^-5^-$	$2^+5^+$
c		c	d		c	d	c	d
	$a_{1g}$	$1^+3^+$	$5^+$	$4^-$	$4^+5^-$	$1^+5^-$	$1^+3^-4^-$	$1^+2^+3^-$
	$a_{1u}$	$1^-3^-$	$5^-$	$4^+$	$4^-5^+$	$1^-5^+$	$1^-3^+4^+$	$1^-2^-3^+$
	$b_{2g}$	$2^+3^+$	$4^+$	$5^-$	$3^+5^-$	$2^+5^-$	$2^+3^-4^-$	$1^+2^+4^-$
	$b_{2u}$	$2^-3^-$	$4^-$	$5^+$	$3^-5^+$	$2^-5^+$	$2^-3^+4^+$	$1^-2^-4^+$
	$a_{2g}$	$4^+$	$2^+3^+$	$1^-3^-$	$2^+3^-4^-$	$1^-2^-3^+$	$3^+5^-$	$1^-5^+$
	$a_{2u}$	$4^-$	$2^-3^-$	$1^+3^+$	$2^-3^+4^+$	$1^+2^+3^-$	$3^-5^+$	$1^+5^-$
	$b_{1g}$	$5^+$	$1^+3^+$	$2^-3^-$	$1^+3^-4^-$	$1^-2^-4^+$	$4^+5^-$	$2^-5^+$
d	$b_{1u}$	$5^-$	$1^-3^-$	$2^+3^+$	$1^-3^+4^+$	$1^+2^+4^-$	$4^-5^+$	$2^+5^-$
	$e_g$	$4^+5^+$	$4^+5^+$	$4^-5^-$	$1^-2^-5^+5^-$	$3^-4^-5^+5^-$	$1^-2^-5^+5^-$	$3^+4^+5^+5^-$
	$e_u$	$4^-5^-$	$4^-5^-$	$4^+5^+$	$1^+2^+5^-5^+$	$3^+4^+5^-5^+$	$1^+2^+5^-5^+$	$3^-4^-5^-5^+$

In  $(aaa)$  units:  $a_1(100)$ ,  $a_2(010)$ ,  $a_3(001)$

$Q: O_h(m\bar{3}m) - a(000)$ ,  $b(1/2, 1/2, 1/2)$ ;

$D_{4h}(4/mmm) - c(1/2, 1/2, 0)$ ,  $d(1/2, 0, 0)$

In  $(2\pi/a, 2\pi/a, 2\pi/a)$  units:  $b_1(100)$ ,  $b_2(010)$ ,  $b_3(001)$

$K: O_h - \Gamma(000)$ ,  $R(1/2, 1/2, 1/2)$ ;  $D_{4h} - M(1/2, 1/2, 0)$ ,  $X(1/2, 0, 0)$

# Induced site-symmetry representations of space group: P m -3 m (221)

with K-vector: G=(0, 0, 0)

[ Symmetry elements Space Group P m -3 m (221) ]

WP	Representative
1a	0,0,0

## Site Symmetry Group

Site Symmetry Group for 0,0,0 (1a)

Site Symmetry - Opera

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry x

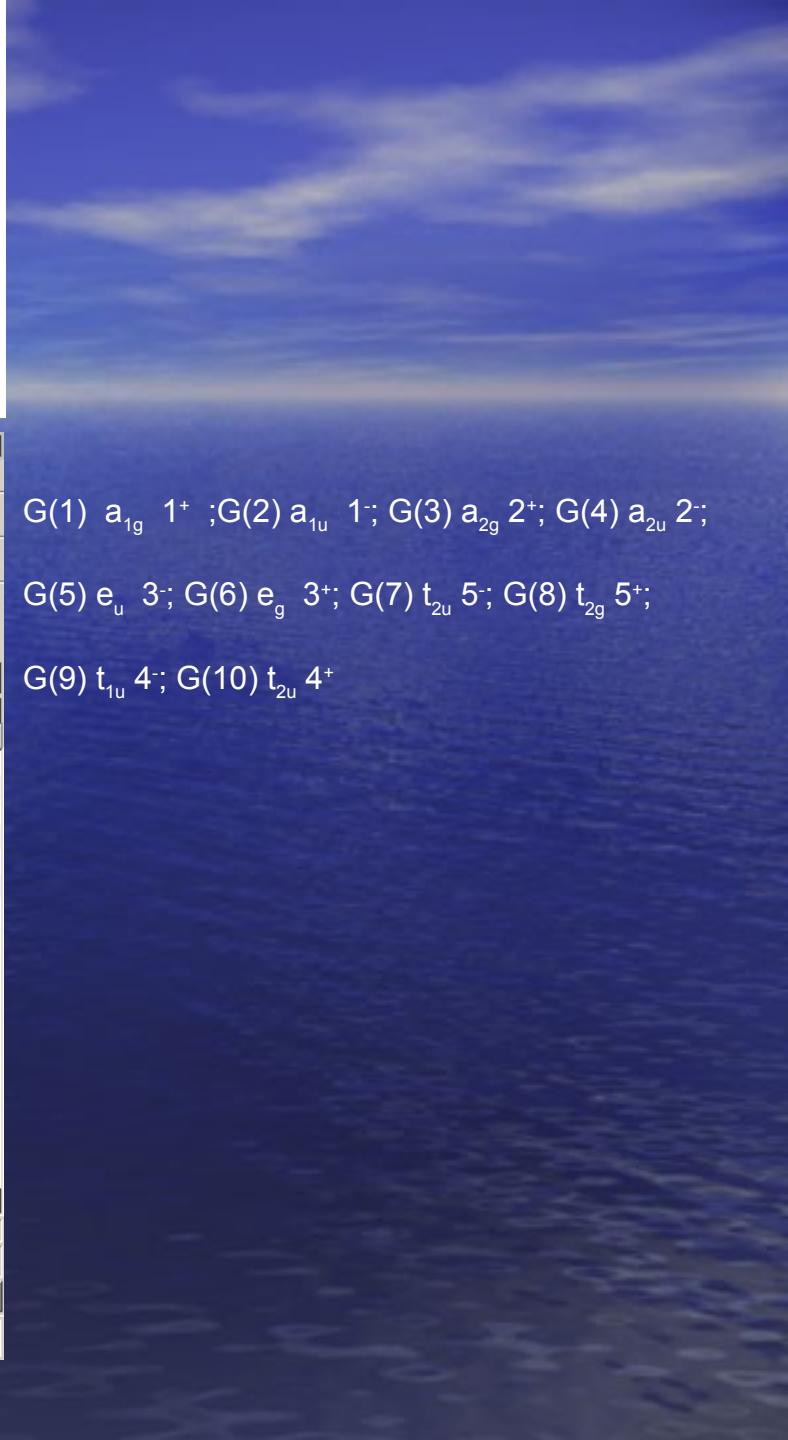
http://www.cryst.ehu.es/cgi-bin/rep/program ? 66

Find in page Find next Voice Author mode Show images Fit to width 100% Be

Induced representations for the point G of P m -3 m

Irreps\Reps	(*G)(1)	(*G)(2)	(*G)(3)	(*G)(4)	(*G)(5)	(*G)(6)	(*G)(7)	(*G)(8)	(*G)(9)	(*G)(10)
A <sub>1g</sub>	1	.	.	.	.	.	.	.	.	.
A <sub>1u</sub>	.	1	.	.	.	.	.	.	.	.
A <sub>2g</sub>	.	.	1	.	.	.	.	.	.	.
A <sub>2u</sub>	.	.	.	1	.	.	.	.	.	.
E <sub>g</sub>	.	.	.	.	1	.	.	.	.	.
E <sub>u</sub>	.	.	.	.	1	.	.	.	.	.
T <sub>2u</sub>	.	.	.	.	.	1	.	.	.	.
T <sub>2g</sub>	.	.	.	.	.	.	1	.	.	.
T <sub>1u</sub>	.	.	.	.	.	.	.	1	.	.
T <sub>1g</sub>	.	.	.	.	.	.	.	.	1	.

Пуск Site S... Until... Total... Micros... EN 10:08 AM



**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

**Induced site-symmetry representations of space group: P m -3 m (221)**

**with K-vector: R=(0.5, 0.5, 0.5)**

[ Symmetry elements Space Group P m -3 m (221) ]

WP	Representative
1a	0,0,0

**Site Symmetry Group**

**Site Symmetry Group for 0,0,0 (1a)**

g <sub>1</sub>	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
----------------	-------	--

**Site Symmetry - Opera**

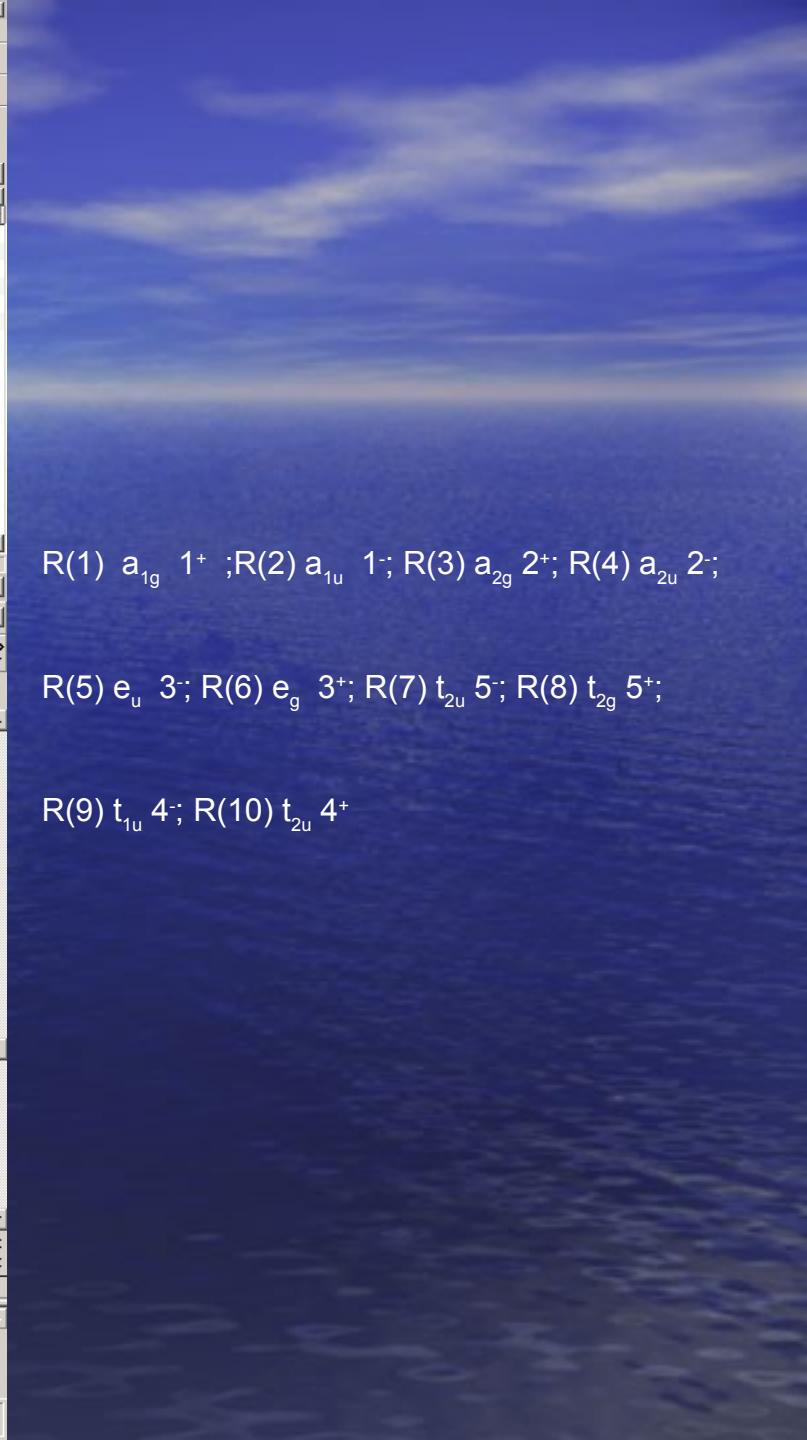
File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

**Induced representations for the point R of P m -3 m**

Irreps\Reps	(*R)(1)	(*R)(2)	(*R)(3)	(*R)(4)	(*R)(5)	(*R)(6)	(*R)(7)	(*R)(8)	(*R)(9)	(*R)(10)
A <sub>1g</sub>	1	.	.	.	.	.	.	.	.	.
A <sub>1u</sub>	.	1	.	.	.	.	.	.	.	.
A <sub>2g</sub>	.	.	1	.	.	.	.	.	.	.
A <sub>2u</sub>	.	.	.	1	.	.	.	.	.	.
E <sub>g</sub>	.	.	.	.	.	1	.	.	.	.
E <sub>u</sub>	.	.	.	.	1	.	.	.	.	.
T <sub>2u</sub>	.	.	.	.	.	.	1	.	.	.
T <sub>2g</sub>	.	.	.	.	.	.	.	1	.	.
T <sub>1u</sub>	.	.	.	.	.	.	.	.	1	.
T <sub>1g</sub>	.	.	.	.	.	.	.	.	.	1

Пуск > Site S... Until... Total... Micros... EN 10:12 AM



**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

http://www.cryst.ehu.es/cgi-bin/rep/program ? G G G G G G Google 68

Find in page Find next Voice Author mode Show images Fit to width 100%

## Induced site-symmetry representations of space group: P m -3 m (221)

with K-vector: R=(0.5, 0.5, 0.5)

[ Symmetry elements Space Group P m -3 m (221) ]

WP	Representative
1b	1/2,1/2,1/2

### Site Symmetry Group

#### Site Symmetry Group for 1/2,1/2,1/2 (1b)

g <sub>1</sub>	x,y,z	( $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ )
		-1 0 0 1
		n n n n

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

http://www.cryst.ehu.es/cgi-bin/rep/program ? G G G G G G Google 68

Find in page Find next Voice Author mode Show images Fit to width 100%

## Induced representations for the point R of P m -3 m

Irreps\Reps	(*R)(1)	(*R)(2)	(*R)(3)	(*R)(4)	(*R)(5)	(*R)(6)	(*R)(7)	(*R)(8)	(*R)(9)	(*R)(10)
A <sub>1g</sub>	.	.	.	1	.	.	.	.	.	.
A <sub>1u</sub>	.	.	1	.	.	.	.	.	.	.
A <sub>2g</sub>	.	1	.	.	.	.	.	.	.	.
A <sub>2u</sub>	1	.	.	.	.	.	.	.	.	.
E <sub>g</sub>	.	.	.	.	1	.	.	.	.	.
E <sub>u</sub>	.	.	.	.	.	1	.	.	.	.
T <sub>2u</sub>	.	.	.	.	.	.	.	.	.	1
T <sub>2g</sub>	.	.	.	.	.	.	.	.	1	.
T <sub>1u</sub>	.	.	.	.	.	.	.	1	.	.
T <sub>1g</sub>	.	.	.	.	.	.	1	.	.	.

Site Symmetry - Opera

New tab Site Symmetry

Find in page Find next Voice Author mode Show images Fit to width 100%

## Induced site-symmetry representations of space group: P m -3 m (221)

with K-vector: R=(0.5, 0.5, 0.5)

[ Symmetry elements Space Group P m -3 m (221) ]

Site Symmetry - Opera

New tab Site Symmetry

File Edit View Bookmarks Widgets Feeds Mail Tools Help

Find in page Find next Voice Author mode Show images Fit to width 100%

Induced representations for the point R of P m -3 m

Irreps\Reps	(*R)(1)	(*R)(2)	(*R)(3)	(*R)(4)	(*R)(5)	(*R)(6)	(*R)(7)	(*R)(8)	(*R)(9)	(*R)(10)
A <sub>1g</sub>	.	.	.	.	.	.	.	.	1	.
A <sub>2g</sub>	.	1	.	.	1	.	.	.	.	.
B <sub>1g</sub>	.	.	.	1	1	.	.	.	.	.
B <sub>2g</sub>	.	.	.	.	.	.	1	.	.	.
E <sub>g</sub>	.	.	.	.	.	.	1	.	1	.
A <sub>1u</sub>	.	.	.	.	.	.	.	.	.	1
A <sub>2u</sub>	1	.	.	.	.	1	.	.	.	.
B <sub>1u</sub>	.	.	1	.	.	1	.	.	.	.
B <sub>2u</sub>	.	.	.	.	.	.	.	1	.	.
E <sub>u</sub>	.	.	.	.	.	.	.	1	.	1

Site Symmetry - Opera

New tab Site Symmetry

File Edit View Bookmarks Widgets Feeds Mail Tools Help

Find in page Find next Voice Author mode Show images Fit to width 100%

Mult.	-	1	1	2	2	2	1	1	2	2	2	.
A <sub>1g</sub>	$\Gamma_1^+$	1	1	1	1	1	1	1	1	1	1	$x^2+y^2+z^2$
A <sub>2g</sub>	$\Gamma_2^+$	1	1	1	-1	-1	1	1	1	-1	-1	$J_z$
B <sub>1g</sub>	$\Gamma_3^+$	1	1	-1	1	-1	1	1	-1	1	-1	$x^2-y^2$
B <sub>2g</sub>	$\Gamma_4^+$	1	1	-1	-1	1	1	1	-1	-1	1	$xy$
E <sub>g</sub>	$\Gamma_5^+$	2	-2	0	0	0	2	-2	0	0	0	$(xz,yz),(J_x,J_y)$
A <sub>1u</sub>	$\Gamma_1^-$	1	1	1	1	1	-1	-1	-1	-1	-1	.
A <sub>2u</sub>	$\Gamma_2^-$	1	1	1	-1	-1	-1	-1	1	1	1	$z$
B <sub>1u</sub>	$\Gamma_3^-$	1	1	-1	1	-1	-1	1	-1	1	1	.
B <sub>2u</sub>	$\Gamma_4^-$	1	1	-1	-1	1	-1	-1	1	1	-1	.
E <sub>u</sub>	$\Gamma_5^-$	2	-2	0	0	0	-2	2	0	0	0	$(xy)$

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

Find in page Find next Voice Author mode Show images Fit to width 100%

Induced site-symmetry representations of space group: P m -3 m (221)

with K-vector: M=(0.5, 0.5, 0)

[Symmetry elements Space Group P m -3 m (221)]

WP	Representative
3d	1/2,0,0

**Site Symmetry Group**

Site Symmetry Group for 1/2,0,0 (3d)

g <sub>1</sub>	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
g <sub>2</sub>	-x+1,-y,z	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

Find in page Find next Voice Author mode Show images Fit to width 100%

Irreps\Reps	(*M)(1)	(*M)(2)	(*M)(3)	(*M)(4)	(*M)(5)	(*M)(6)	(*M)(7)	(*M)(8)	(*M)(9)	(*M)(10)
A <sub>1g</sub>	1	.	.	.	.	.	.	.	1	.
A <sub>2g</sub>	.	1	1	.	.	1	.	.	.	.
B <sub>1g</sub>	.	1	.	.	.	1	1	.	.	.
B <sub>2g</sub>	.	.	.	.	1	.	.	1	.	.
E <sub>g</sub>	.	.	.	1	.	.	.	1	1	1
A <sub>1u</sub>	.	1	.	.	.	.	.	.	.	1
A <sub>2u</sub>	1	.	.	1	1	.	.	.	.	.
B <sub>1u</sub>	1	.	.	.	.	1	.	1	.	.
B <sub>2u</sub>	.	.	.	.	.	.	1	.	.	1
E <sub>u</sub>	.	.	1	.	.	.	1	1	1	1

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

Find in page Find next Voice Author mode Show images Fit to width 100%

Irreps	1	1	2	2	1	1	2	2	2	
A <sub>1g</sub>	$\Gamma_1^+$	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$
A <sub>2g</sub>	$\Gamma_2^+$	1	1	1	-1	-1	1	1	-1	$J_z$
B <sub>1g</sub>	$\Gamma_3^+$	1	1	-1	1	-1	1	-1	1	$x^2-y^2$
B <sub>2g</sub>	$\Gamma_4^+$	1	1	-1	-1	1	1	-1	-1	$xy$
E <sub>g</sub>	$\Gamma_5^+$	2	-2	0	0	0	2	-2	0	$(xz,yz),(J_x,J_y)$
A <sub>1u</sub>	$\Gamma_1^-$	1	1	1	1	1	-1	-1	-1	.
A <sub>2u</sub>	$\Gamma_2^-$	1	1	1	-1	-1	-1	-1	1	$z$
B <sub>1u</sub>	$\Gamma_3^-$	1	1	-1	1	-1	-1	1	-1	.
B <sub>2u</sub>	$\Gamma_4^-$	1	1	-1	-1	1	-1	1	1	.
E <sub>u</sub>	$\Gamma_5^-$	2	-2	0	0	0	-2	2	0	$(x,y)$

Пуск Site Sym... Total Com... Microsoft P... 11:18 AM

M(1) 1<sup>+</sup>; M(2) 1<sup>-</sup>; M(3) 3<sup>+</sup>; M(4) 3<sup>-</sup>; M(5) 2<sup>+</sup>; M(6) 2<sup>-</sup>

M(7) 4<sup>+</sup>; M(8) 4<sup>-</sup>; M(9) 5<sup>-</sup>; M(10) 5<sup>+</sup>

## Anatase TiO<sub>2</sub>

Ti	a	D <sub>2d</sub>	s, z <sup>2</sup>	a <sub>1</sub>
			x <sup>2</sup> -y <sup>2</sup>	b <sub>1</sub>
			z, xy	b <sub>2</sub>
			(x, y), (xz, yz)	e

Point Group Tables - Opera

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Point Group Tables x

http://www.crys.ehu.es/cgi-bin/rep/programs/sa ? G G G G G Google

Find in page Find next Voice Author mode Show images Fit to width 100% 60

Bilbao Crystallographic Server → Point Group Tables Help

### Point Group Tables of D<sub>2d</sub>(-42m)

Character Table

D <sub>2d</sub> (-42m)	#	1	2	-1	2 <sub>x</sub>	m <sub>d</sub>	functions
Mult.	-	1	1	2	2	2	.
A <sub>1</sub>	Γ <sub>1</sub>	1	1	1	1	1	x <sup>2</sup> +y <sup>2</sup> ,z <sup>2</sup>
A <sub>2</sub>	Γ <sub>2</sub>	1	1	1	-1	-1	J <sub>z</sub>
B <sub>1</sub>	Γ <sub>3</sub>	1	1	-1	1	-1	x <sup>2</sup> -y <sup>2</sup>
B <sub>2</sub>	Γ <sub>4</sub>	1	1	-1	-1	1	z,xy
E	Γ <sub>5</sub>	2	-2	0	0	0	(xy),(xz,yz),(J <sub>x</sub> ,J <sub>y</sub> )

[ List of irreducible representations in matrix form ]

Flyck Total Com... symnloca... Point Gro... EN 3:39 PM

O	e	C <sub>2v</sub>	s, z, z <sup>2</sup>	a <sub>1</sub>
			xy, x <sup>2</sup> -y <sup>2</sup>	a <sub>2</sub>
			x, xz	b <sub>1</sub>
			y, yz	b <sub>2</sub>

## Point Group Tables - Opera

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Point Group Tables X

Back Forward Stop Refresh <http://www.crys.ehu.es/cgi-bin/repl/programs/sa> ? G G G G G G G Google 60

Find in page Find next Voice Author mode Show images Fit to width 100%

Bilbao Crystallographic Server → Point Group Tables

Help

## Point Group Tables of $C_{2v}(\text{mm}2)$

Character Table

$C_{2v}(\text{mm}2)$	#	1	$2_z$	$m_y$	$m_x$	functions
$A_1$	$\Gamma_1$	1	1	1	1	$z, x^2, y^2, z^2$
$A_2$	$\Gamma_3$	1	1	-1	-1	$xy, J_z$
$B_1$	$\Gamma_2$	1	-1	1	-1	$x, xz, J_y$
$B_2$	$\Gamma_4$	1	-1	-1	1	$y, yz, J_x$

[ List of irreducible representations in matrix form ]

**Table 4.21.** Simple induced representations of the  $D_{4h}^{19} - I4_1/AMD$  space group

$q$	$\beta$	$\Gamma$	$M$	$X$	$P$	$N$
					$a$ $b$	
$a$	$a_1$	$1^+ 4^-$	1	1	2      1	$1^+ 2^-$
	$a_2$	$3^+ 2^-$	2	2	2      1	$1^- 2^+$
$b$	$b_2$	$3^- 2^+$	1	2	1      2	$1^+ 2^-$
	$b_1$	$1^- 4^+$	2	1	1      2	$1^- 2^+$
	$e$	$5^+ 5^-$	3 4	1 2	1 2      1 2	$1^+ 1^- 2^+ 2^-$
						$c$ $d$
$c$	$a_g$	$1^+ 2^+ 5^+$	1 4	1 2	1 2	$1^+ 1^+ 2^+ 2^-$ $1^+ 1^- 2^- 2^-$
	$a_u$	$1^- 2^- 5^-$	2 4	1 2	1 2	$1^- 1^- 2^+ 2^-$ $1^+ 1^- 2^+ 2^+$
$d$	$b_g$	$3^+ 4^+ 5^+$	2 3	1 2	1 2	$1^+ 1^- 2^+ 2^+$ $1^- 1^- 2^+ 2^-$
	$b_u$	$3^- 4^- 5^-$	1 3	1 2	1 2	$1^+ 1^- 2^- 2^-$ $1^+ 1^+ 2^+ 2^-$

The basis translations vectors of direct and reciprocal lattices and coordinates of  $K$ -set points are in Table 4.20.

$Q: D_{2d}(4m2) - a(000), b(1/2, 1/2, 0)$

$C_{2h}(.2/m.) - c(3/8, 1/8, 1/4), d(-1/8, -3/8, 1/4)$ .

**Site Symmetry - Opera**

New tab Site Symmetry

File Edit View Bookmarks Widgets Feeds Mail Tools Help

Find in page Find next Voice Author mode Show images Fit to width 100%

Find in page Find next Voice Author mode Show images Fit to width 100%

Find in page Find next Voice Author mode Show images Fit to width 100%

Induced site-symmetry representations of space group: I 4<sub>1</sub>/a m d (141)

with K-vector: M=(0.5, 0.5, 0.5)

[ Symmetry elements Space Group I 4<sub>1</sub>/a m d (141) ]

WP	Representative
8e	0,1/4,z

**Site Symmetry Group**

Site Symmetry Group for 0,1/4,z (8e)

g <sub>1</sub>	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
----------------	-------	---

Induced representations for the point M of I 4<sub>1</sub>/a m d

Irreps\Reps	(^M)(1)	(^M)(2)	(^M)(3)	(^M)(4)
A <sub>1</sub>	.	.	2	.
A <sub>2</sub>	.	.	.	2
B <sub>1</sub>	1	1	.	.
B <sub>2</sub>	1	1	.	.

Bilbao Crystallographic Server  
http://www.cryst.ehu.es

For comments, please mail to  
cryst@wm.ic.ehu.es

M(1) 3; M(2) 4; M(3) 1 ; M(4) 2

**Site Symmetry - Opera**

New tab Site Symmetry

File Edit View Bookmarks Widgets Feeds Mail Tools Help

Find in page Find next Voice Author mode Show images Fit to width 100%

Find in page Find next Voice Author mode Show images Fit to width 100%

Find in page Find next Voice Author mode Show images Fit to width 100%

**Irreducible representations**

Character table for mm2

C <sub>2y</sub> (mm2)	#	1	2 <sub>z</sub>	m <sub>y</sub>	m <sub>x</sub>	functions
A <sub>1</sub>	Γ <sub>1</sub>	1	1	1	1	z,x <sup>2</sup> ,y <sup>2</sup> ,z <sup>2</sup>
A <sub>2</sub>	Γ <sub>3</sub>	1	1	-1	-1	xy,J <sub>z</sub>
B <sub>1</sub>	Γ <sub>2</sub>	1	-1	1	-1	x,xz,J <sub>y</sub>
B <sub>2</sub>	Γ <sub>4</sub>	1	-1	-1	1	y,yz,J <sub>x</sub>

**Subduced representations**

Character table for the subduced representations (^M↓mm2) for Wyckoff position 8e

Пуск Microsoft PowerPoint ... Site Symmetry - Opera EN 11:30 AM

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

Find in page Find next Voice Author mode Show images Fit to width 100%

## Induced site-symmetry representations of space group: $I\bar{4}_1/a\text{ m d}$ (141)

with K-vector:  $\mathbf{N}=(0, 0.5, 0)$

[Symmetry elements Space Group  $I\bar{4}_1/a\text{ m d}$  (141)]

WP	Representative
4a	0,3/4,1/8

**Site Symmetry Group**

Site Symmetry Group for 0,3/4,1/8 (4a)

$g_1$	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
-------	-------	--

**Irreducible representations**

Character table for -42m

$D_{2d}(-42m)$	#	1	2	-4	$2_x$	$m_d$	functions
Mult.	-	1	1	2	2	2	.
$A_1$	$\Gamma_1$	1	1	1	1	1	$x^2+y^2, z^2$
$A_2$	$\Gamma_2$	1	1	1	-1	-1	$J_z$
$B_1$	$\Gamma_3$	1	1	-1	1	-1	$x^2-y^2$
$B_2$	$\Gamma_4$	1	1	-1	-1	1	$z, xy$
$E$	$\Gamma_5$	2	-2	0	0	0	$(x,y), (xz,yz), (J_x, J_y)$

**Subduced representations**

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry

Find in page Find next Voice Author mode Show images Fit to width 100%

$(^N)(4)$

## Induced representations

Induced representations for the point N of  $I\bar{4}_1/a\text{ m d}$

Irreps\Reps	$(^N)(1)$	$(^N)(2)$	$(^N)(3)$	$(^N)(4)$
$A_1$	.	1	1	.
$A_2$	1	.	.	1
$B_1$	1	.	.	1
$B_2$	.	1	1	.
$E$	1	1	1	1

Bilbao Crystallographic Server  
<http://www.cryst.ehu.es>

For comments, please mail to [cryst@wm.lc.ehu.es](mailto:cryst@wm.lc.ehu.es)

Пуск Microsoft P... Site Sym... Документ... 11:40 AM

N(1) 1<sup>+</sup>; N(2) 1<sup>-</sup>; N(3) 2<sup>+</sup>; N(4) 2<sup>-</sup>

**Site Symmetry - Opera**

File Edit View Bookmarks Widgets Feeds Mail Tools Help

New tab Site Symmetry <http://www.cryst.ehu.es/cgi-bin/rep/programs/sit> Google

Find in page Find next Voice Author mode Show images Fit to width 100%

## Induced site-symmetry representations of space group: I 4<sub>1</sub>/a m d (141)

with K-vector: N=(0, 0.5, 0)

[ Symmetry elements Space Group I 4<sub>1</sub>/a m d (141) ]

WP	Representative
8e	0,1/4,z

**Site Symmetry Group**

**Irreducible representations**

Character table for mm2

C <sub>2h</sub> (mm2)	#	1	2 <sub>z</sub>	m <sub>y</sub>	m <sub>x</sub>	functions
A <sub>1</sub>	Γ <sub>1</sub>	1	1	1	1	z,x <sup>2</sup> ,y <sup>2</sup> ,z <sup>2</sup>
A <sub>2</sub>	Γ <sub>3</sub>	1	1	-1	-1	xy,J <sub>z</sub>
B <sub>1</sub>	Γ <sub>2</sub>	1	-1	1	-1	x,xz,J <sub>y</sub>
B <sub>2</sub>	Γ <sub>4</sub>	1	-1	-1	1	y,yz,J <sub>x</sub>

**Subduced representations**

Character table for the subduced representations ( $\Gamma \downarrow mm2$ ) for Wyckoff

**Induced representations**

Induced representations for the point N of I 4<sub>1</sub>/a m d

Irreps\Reps	( $\Gamma$ )(1)	( $\Gamma$ )(2)	( $\Gamma$ )(3)	( $\Gamma$ )(4)
A <sub>1</sub>	.	2	2	.
A <sub>2</sub>	2	.	.	2
B <sub>1</sub>	1	1	1	1
B <sub>2</sub>	1	1	1	1

N(1) 1<sup>+</sup>; N(2) 1<sup>-</sup>; N(3) 2<sup>+</sup>; N(4) 2<sup>-</sup>

The background of the image is a wide-angle photograph of a seascape. The water is a deep, vibrant blue, with small, gentle ripples across its surface. Above the horizon, the sky is a lighter shade of blue, dotted with wispy, white clouds. In the upper left corner, there is a soft, glowing effect that suggests either a sunrise or sunset, with hints of yellow and orange visible.

**THANKS FOR ATTENTION!**